

Expansion of generalized polynomial set $\mathcal{Q}_m^k\{(x_n), y\}$ in terms of Jocobi and Sister Celin's Polynomials.

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Abstract :

Recently, we introduced “An unification of certain generalized polynomials set $Q_m^k\{(x_n), y\}$ with the help of generating relation which contains the generalized Lauricella functions in the notation of Burchnall and Chaundy [2]. It is shown that this polynomial, set happen to be a generalization of as many as forty orthogonal and non-orthogeenal polynomials. In this paper, we introduced the polynomials set $Q_m^k\{(x_n), y\}$ in terms Jocobi and Sister Celin’s polynomials.

Introduction :

The generalized polynomial $\mathcal{Q}_m^k\{(x_n), y\}$ set is defined by means of generating relation.

where $\mu, \mu_1, \mu_2, \dots, \mu_n$ are real and $e, e_1, e_2, e_3, \dots, e_n$ are non-negative integers.

The left hand side of (1.1) contains the product of generalized hypergeometric function and Lauricella function in the notation of Burchnall and chaundy. The polynomial set contains a number of parameters for simplicity. It is denoted by $Q_m^k\{(x_n), y\}$ where m is the order of the polynomial set.

(1.2) Notations

1. $(n) = 1, 2, 3, \dots, n$
2. $(A_p) = A_1, A_2, \dots, A_p$
3. $(A_p)_m = (A_1)_m, (A_2)_m, (A_3)_m, \dots, (A_p)_m$
4. $(A_p)_m = (A_p)_n, (A_2)_n, (A_3)_n, \dots, (A_p)_n$
5. $[(A_p), i] = A_1, A_2, \dots, A_{i-1}, \dots, A_p$
6. $[(a, b)] = \prod_{r=1}^a \left(\frac{b+r+1}{a} \right)_k = \left(\frac{b}{a} \right)_k \left(\frac{b+1}{a} \right)_k \dots \left(\frac{b+a-1}{a} \right)_k$

where an empty product is treated as unity

Theorem (1) for $e_2 > 1, \dots, e_n > 1$, we have

$$\begin{aligned} Q_m^k \{ {}^x n, y \} &= K \sum_{i=0}^m \frac{(m+c)!(-1)^i (2i+c+d+1)}{(n-i)!(c+d+i+1)_{m+1}} P_i^{(c,d)} (1+2x^e) \\ &\times F_{P+r+k; v_1, v_2, \dots, v_m}^{q+s+h+1; u_1, u_2, u_3, \dots, u_n} \left[\begin{matrix} [(-m+i); e_1 e_2 \dots e_n] \\ [(-m+c); e_1, e_2, \dots, e_n] \end{matrix} \right] \\ &[1 - (B_s) - m; e_1, e_2 - 1, \dots, (e_n - 1)], [1 - (c_p) - m; e_1, e_2 e_3, \dots, e_n] \\ &[(1 - (A_r) - m); e_1, e_2 - 1, \dots, (e_m - 1)], [(1 - (D_q) - m); e_1 e_2 e_3, \dots, e_n] \\ &[(-c - d - i - m - 1); e_1, e_2, \dots, e_n] [(a_h) : 1], [(\infty_{u_1})], [(\infty_{u_2}) : 1], \dots, [(\infty_{u_n}) : 1] \\ &[(p_k) : 1], [(\alpha_{v_1}) : 1], [(\alpha_{v_2}) : 1], \dots, [(\alpha_{v_n}) : 1] \\ &\frac{\mu_1 (-1)^{e_1(r+s+p-q+1)}}{\mu^e}, \frac{\mu_2 (-1) x_{2e_2}^{e_2(r+s+p-q+1)+r+s}}{\mu^{e_2}}, \dots, \frac{\mu_n (-1) x_{n e_n}^{e_n(r+s+p-q+1)+r+s}}{\mu^{e_n}} \end{aligned}$$

$$\text{where } K = \frac{[(A_r)_m [(C_p)]_m (\mu x^e)^m}{[(B_s)]_m [(D_q)]_m m!}$$

Proof : After little simplification (1.1) gives

$$\begin{aligned} \sum_{m=0}^{\infty} S_m^k \{ (x_n), y \} t^m &= \sum_{m=0}^{\infty} \sum_{n_1=0, n_2=0, \dots, n_n=0}^{\infty} \frac{[(a_h)] n_1 (\mu_1)^{n_1} [(A_r)]_{m+n_2+\dots+n_n}}{[(b_k)]_{n_1} n_1! y^{e_1 n_1 [(B_s)]_{m+n_2+\dots+n_n}}} \\ &\times \frac{[(C_p)]_n [(\infty_{u_n})]_{e_n} (\mu x^e)^n (\mu_2 x_2^{e_2})^{n_2} \dots (\mu_m x_m^{e_m})^{n_m} t^{m+e_1 n_1 + e_2 n_2 + \dots + e_n n_n}}{[(D_q)]_m [(\alpha_{v_n})] m! n_2! n_n!} \end{aligned}$$

we have

$$\begin{aligned}
& (x^e)^m = n!(m+c)! \sum_{i=0}^m \frac{(-1)^i (2i+c+d+1)}{(n-i)!(c+d+i+1)_{m+1}} p_i^{(c,d)}(1+2x^e) \\
& \sum_{m=0}^{\infty} Q_m^k \{ (x_n), y \} t^m = \sum_{m=0}^{\infty} \sum_{\substack{n_1=0, n_2=0, \dots, n_n=0}} \frac{\left[(A_r)_{m+m_2+\dots+n_n} \left[(C_p) \right]_m \right]}{\left[(B_s) \right]_{m+n_2+\dots+n_n} \left[D_q \right]_m} \\
& \times \frac{\left[(\alpha_{v_1}) \right]_{n_1} \left[(\alpha_{v_2}) \right]_{n_2} \dots \left[(\alpha_{v_n}) \right]_{n_n} \left[(a_h) \right]_{n_1} \mu^m (\mu_1)^{n_1} (\mu_2 x_2^{e_2})^{n_2} \dots (\mu_n x_n^{e_n})^{n_n}}{\left[\alpha_{v_1} \right]_{n_1} \left[(\alpha_{v_2}) \right]_{n_2} \dots \left[\alpha_{v_n} \right]_{n_n} \left[(b_k) \right]_{n_1} m_1! y^{e_1 n_1} \dots n_2! n_n!} \\
& \times \frac{(-1)^i (2i+c+d+1)(n+1)! p_i^{(c,d)}(1+2x^e) t^{m+e_1 n_1 + e_2 n_2 + \dots + e_n n_n}}{(m-i)(c+d+i+1)_{m+1}} \\
& = \sum_{m=0}^{\infty} \sum_{i=0}^m \sum_{n_1=0}^{m/e_1} \frac{\sum_{n_2=0}^{m-e_1 n_1}}{\dots} \dots \sum_{n_n=0}^{[m-e_1 n_1 - e_2 n_2 - \dots - e_{n-1} n_{n-1}]} \dots \dots .2.2 \\
& \times \frac{\left[A_r \right]_{m-e_1 n_1 - (e_2-1)_{n_2} - \dots - (e_{n-1})_{n_n}} \left[C_p \right] m - e_1 n_1 - e_2 n_2 - \dots - e_m n_n \left[(a_n) \right]_{n_1} \left[(\alpha_{v_1}) \right]_{n_1} \left[(\alpha_{v_2}) \right]_{n_2} \dots}{\left[(B_s) \right]_{n-e_1 n_1 - (e_2-1)_{n_2} - \dots - (e_{n-1}-1)_{n_n}} \left[(D_q) \right]_{m-e_1 n_1 - e_2 n_2 - \dots - e_n n_n} \left[(b_k) \right]_{n_1} \left[(\alpha_{v_1}) \right]_{n_1} \left[(\alpha_{v_2}) \right]_{n_2}} \\
& \frac{\left[(\alpha_{v_n}) \right]_{n_n} (\mu_1)^{n_1} (\mu_2 x_2^{e_2})^{n_2} \dots (\mu_n x_n^{e_n})^{n_n}}{\left[(\alpha_{v_m}) \right]_{n_n} m_1! y^{e_1 n_1 - n_2! - \dots - n_n!}} \frac{(-1)^i (2i+c+d+1) \mu^{n-e_1 m_1 - e_2 m_2 - \dots - e_n n_n}}{(m-i - e_1 n_1 - e_2 n_2 - \dots - e_n n_n)!} \\
& \times \frac{(n+c-e_1 n_1 - e_2 n_2 - \dots - (e_n n_n)!) P_i^{(c,d)}(1+2x^e) t^m}{(c+d+i+1)_{m+1} - e_1 n_1 - e_2 n_2 - \dots - e_n n_n} \dots .(2.3)
\end{aligned}$$

Equating the coefficient of t^m from both sides we get if

$$e_2 > 1 \dots e_n > 1$$

$$Q_m^k \{ {}^x n, y \} = K \sum_{i=0}^m \frac{(m+c)! (-1)^i (2i+c+d+1) p_i^{(c,d)} (1+2x^e)}{(m+i)! (c+d+i+1)_{m+1}}$$

$$\times \sum_{n_1, n_2, \dots, n_n=0}^{\infty} \frac{[1-(B_s)-m]_{e_1 n_1 + (e_2 - 1) n_2 + \dots + (e_n - 1) n_n}}{[1-(A_r)-m]_{e_1 n_1 + (e_2 - 1) n_2 + \dots + (e_n - 1) n_n}}$$

$$\times \frac{[1-(D_q)-m]_{e_1 n_1 + e_2 n_2 + \dots + e_n n_n} [(a_h)]_{n_1} [(\infty_{u_1})]_{n_1} [(\infty_{u_2})]_{n_2} \dots [(\infty_{u_l})]_{n_l}}{[1-(c_p)-m]_{e_1 n_1 + \dots + e_2 n_2 + \dots + e_n n_n} [(b_k)]_{n_1} [(\alpha_{v_1})]_{n_1} [(\alpha_{v_2})]_{n_2} \dots [(\alpha_{v_l})]_{n_l}}$$

$$\begin{aligned}
 & \times \frac{(\mu_1)^{n_1} (\mu_2 x_2^{e_2})^{m_2} \dots (\mu_n x_n^{e_n})^{n_n} (-m+i)_{e_1 n_1 + e_2 n_2 + \dots + e_n n_n}}{n_1! y^{e_1 n_1} n_2! \dots n_n! (-n+c)_{e_1 n_1 + e_2 n_2 + \dots + e_n n_n}} \\
 & \times \frac{(\mu_1)^n (\mu_2 x_2^{e_2})^{n_2} \dots (\mu_n x_n^{e_n})^{n_n} (-n+i)_{e_1 n_1 + e_2 n_2 + \dots + e_n n_n} n_n}{n_1! y^{e_1 n_1} n_2! \dots n_n! (-n+c)_{e_1 n_1 + e_2 n_2 + \dots + e_n n_n}} \\
 & \times \frac{(-c-d-i-n-1)_{e_1 n_1 + e_2 n_2 + e_3 n_3 + \dots + e_n n_n} (-1)^{e_1 (r+s+p+q+1)n_1}}{\mu^{e_1 n_1} + e_2 n_2 + \dots + e_n n_n} \\
 & \times \frac{(-1)^{e_2 \{(r+s+p+q+1)+r+s\} m_2} \dots (-1)^{e_n \{(r+s+p+q+1)+r+s\} n_n}}{1} \dots \dots \dots (2.4)
 \end{aligned}$$

where c is non-negative integer

The single terminating factor $(-n+c)_{e_1 n_1 + e_2 n_2 + \dots + e_n n_n}$ make all summation in (2.4) runs up to ∞ .

Hence, the theorem (2.1)

particular cases (2.1)

(i) Hermite Polynomials

If we set $r = 0 = s = p = q = \mu_1 = v_1 = h = k; e_1 = 2 = \mu$

$x_1 = x, x_2 = 1 = e = y; \mu_1 = \sqrt{-4}, \mu_1 = \sqrt{-4}$, we have

$$H_n(x) = \sum_{i=0}^m \frac{2^m (-1)^i (m+c)! (2i+c+d+1)}{(m-i)! (c+d+i+1)_{m+1}} P_i^{(c,d)} (1+2x^e)$$

$$F \left[\begin{matrix} \Delta(2; -m+i), \Delta(2; -c-d-i-m-1); \\ \Delta(2; -m+c) \end{matrix} \middle| -1 \right]$$

(ii) Legendre Polynomials

on making the substitution

$$r = 0 = s = p = q = \mu_1 = v_1;$$

$k = 1 = e = \mu = \mu_1 = y = b_1, e_1 = 2$ and $\frac{x}{\sqrt{x^2-1}}$ for x we set

$$P_m(x) = \sum_{i=0}^m \frac{2^m (-1)^i (m+c)! (2i+c+d+1) P_i^{(c,d)} (1+2x^e)}{(m-i)! (c+d+i+1)_{m+1}}$$

$$\times \, {}_F\left[\begin{matrix} \Delta(2; -m+i), \Delta(2; -c-d-i-m-1); \\ \Delta(2; -m+c), 1; \end{matrix} -1 \right]$$

Similarly, specializing the parameters in (2.1) all the polynomials defined by authours [8] can also be decluced in term of Jocobi also.

Theorem (2) for $e_2 > 1 \dots e_n > 1$, we obtain

$$\begin{aligned}
Q_m^k \{(x_n), y\} &= \frac{k n! \left(\frac{1}{2} \right)_m \left[\left(f_h \right) \right]_m}{\left[\left(d_g \right) \right]_m} \sum_{m=0}^{\infty} \frac{(-1)^{j(2j+1)}}{(m-j)!(n+j+1)!} c_j(x^e) \\
&\times F_{p+r+k; v_1, v_2, \dots, v_n}^{q+s+h+l; u_1, u_2, \dots, u_m} \left[\begin{array}{l} [(-m+j); e_1, e_2, \dots, e_n] [-m-j-1) e_1, e_2, \dots, e_n] \\ [(-m), e_1, e_2, \dots, e_n] \left[\left(\frac{1}{2} - m \right); e_1, e_2, \dots, e_n \right] \end{array} \right] \\
&\left[[(1-(B_s)-m); e_1, e_2-1, \dots, (e_n-1)], [(1-(D_q)-m)e_1, e_2, \dots, e_n] \right] \\
&\left[[(1-(A_r)-m); e_1, e_2-1, \dots, (e_n-1)], [(1-(C_p)-m); e_1, e_2, \dots, e_n] \right] \\
&\left[[(1-(d_2)-m); e_1, e_2, \dots, e_n] [(a_h); 1] [\infty_u, 1] [(\infty_{u_2}); 1], \dots, [(\infty_{u_n}); 1] \right] \\
&\left[[(1-(f_h))-m); e_1, e_2, \dots, e_n] [(b_k); 1] [(\alpha_{v_1}); 1] [(\alpha_{v_2}); 1], \dots, [(\alpha_{v_n}); 1] \right] \\
&\frac{\mu_1 (-1)^{e_1(r+s+p+q+f'+h'+1)}}{\mu^{e_1} y^{e_1}} \cdot \frac{\mu_2 x_2^{e_2} (-1)^{e_2(r+s+p+q+f'+h'+1)+r+s}}{\mu^{e_2}} \cdot \dots \cdot \\
&\frac{\mu_n x_n^{e_n} (-1)^{e_n(r+s+p+q+f'+h'+1)+r+s}}{\mu^{e_n}} \quad \dots \quad 3.1
\end{aligned}$$

Proof

$$(x^e)^n = \frac{(n!)^2}{(d_g^{'})_m} \left(\frac{1}{2} \right)_m \left[(f_h^{'})_m \right] \sum_{j=0}^m \frac{(-1)^j (2j+1)}{(m-j)!(n+j+1)!} e_j(x^e)$$

putting the value of $(x^e)^m$ in equation (2.2) we get

$$\sum_{m=0}^{\infty} Q_m^k \{ (x_n), y \} t^m = \sum_{m=0}^{\infty} \sum_{n_1=0, n_2=0, \dots, n_p=0}^{\infty} \sum_{j=0}^m \frac{[(A_r)]_{m+n_2+\dots+n_p} [(C_p)]_m}{[(B_s)]_{m+n_2+\dots+n_p} [(D_q)]_m}$$

$$\begin{aligned}
 & \times \frac{\left[(\infty_{u_1}) \right]_{n_1} \left[(\infty_{u_2}) \right]_{n_2} \dots \left[(\infty_{u_n}) \right]_{n_n} \left[(a_h) \right]_{n_1} \mu^m (\mu_1)^{n_1} \left(\mu_2 x_2^{e_2} \right)^{n_2} \dots \left(\mu_n x_n^{e_n} \right)^{n_n}}{\left[(\alpha_{v_1}) \right]_{n_1} \left[(\alpha_{v_2}) \right]_{n_2} \dots \left[(\alpha_{v_n}) \right]_{n_n} \left[(b_k) \right]_{n_1} n_1! y^{e_1 n_1} \frac{n_2!}{n_2!} \frac{m_n!}{m_n!}} \\
 & \times \frac{(-1)^j (2j+1) \left(\frac{1}{2} \right)_m n! \left[(f'_h) \right]_m t^{m+e_1 n_1 + e_2 n_2 + \dots + e_n n_n}}{(n-j)! (n+j+1)! \left[(d'_g) \right]_m} \\
 & = \sum_{m=0}^{\infty} \sum_{j=0}^m \sum_{n_1=0}^{e_1} \sum_{n_2=0}^{e_2} \dots \left[\frac{m - e_1 n_1 - e_2 n_2 - \dots - e_{n-1} n_{n-1}}{\sum_{n_n=0}^{e_n}} \right] \\
 & \times \frac{\left[(A_r) \right]_{m-e_1 n_1} - (e_2 - 1)_{n_2} \dots (e_n - 1)_{n_n} \left[(C_p) \right]_{m-e_1 n_1 - e_2 n_2 - \dots - e_n n_n}}{\left[(B_s) \right]_{m-e_1 n_1 - (e_2 - 1)_{n_2}} \dots (e_n - 1)_{n_n} [(Dq)]_{m-e_1 n_1 - e_2 n_2 - \dots - e_n n_n}} \\
 & \times \frac{\left[(a_h) \right]_{n_1} \left[(\infty_{u_1}) \right]_{n_1} \left[(\infty_{u_2}) \right]_{n_2} \dots \left[(\infty_{u_n}) \right]_{n_n} \times (\mu_1)^{n_1} (\mu_2 x_2^{e_2})^{n_2} \dots (\mu_n x_n^{e_n})^{n_n}}{\left[(b_k) \right]_{n_1} \left[(\alpha_{v_1}) \right]_{n_1} \left[(\alpha_{v_2}) \right]_{n_2} \dots \left[(\alpha_{v_n}) \right]_{n_n} n_1! y^{e_1 n_1} n_2! \dots n_n!} \\
 & \times \frac{(-1)^j (2j+1) \left(\frac{1}{2} \right)_{m-e_1 n_1 - e_2 n_2 - \dots - e_n n_n} \mu^{n - e_1 n_1 - e_2 n_2 - \dots - e_n n_n}}{(m_{-j-e_1 n_1 - e_2 n_2} - \dots - e_n n_n)!} \\
 & \frac{(n - e_1 n_1 - e_2 n_2 - \dots - e_n n_n)!}{(n+j+1 - e_1 n_1 - e_2 n_2 - \dots - e_n n_n)!} \\
 & \times \frac{\left[(f'_h) \right]_{m-e_1 n_1 - e_2 n_2 - \dots - e_n n_n} c_j(x^e) t^m}{\left[(d'_g) \right]_{m-e_1 n_1 - e_2 n_2 - \dots - e_n n_n}} \dots \dots \dots (3.2)
 \end{aligned}$$

Equating the coefficient of t^m from both side we get

$$\begin{aligned}
 Q_m^k \{ (x_n), y \} &= \frac{k \left(\frac{1}{2} \right)_m \left[(f'_h) \right]_m m!}{\left[(d'_g) \right]_m} \sum_{j=0}^{\infty} \frac{(-1)^j (2j+1) c_j(x^e)}{(n-j)! (n+j+1)!} \\
 &\times \sum_{n_1, n_2, n_3, \dots, n_n=0}^{\infty} \frac{[1 - (B_s) - m]_{e_1 n_1 + (e_2 - 1)_{n_2} + \dots + (e_n - 1) n_n}}{[1 - (A_r) - m]_{e_1 n_1} + (e_2 - 1)_{n_2} + \dots + (e_{n-1}) n_n} \\
 &\times \frac{[1 - (D_q) - m]_{e_1 n_1 + p_2 n_2 + \dots + e_n n_n} [1 - (d'_g) - m]_{m+e_1 n_1 + e_2 n_2 + \dots + e_n n_n}}{[1 - (C_p) - m]_{e_1 n_1 + p_2 n_2 + \dots + e_n n_n} \cdot [1 - (f'_h) - m]_{m+e_1 n_1 + e_2 n_2 + \dots + e_n n_n}}
 \end{aligned}$$

$$\begin{aligned} & \times \frac{[(-m+j)_{e_1 n_1 + e_2 n_2 + \dots + e_n n_n} (-m-j-1)_{e_1 n_1 + e_2 n_2 + \dots + e_n n_n}]}{\left(\frac{1}{2}-m\right)_{e_1 n_1 + e_2 n_2 + \dots + e_n n_n} (-m)_{e_1 n_1 + e_2 n_2 + \dots + e_n n_n}} \\ & \times \frac{[(a_h)]_{m_1} [(\infty_{u_1})]_{n_1} [(\infty_{u_2})]_{n_2} \dots [(\infty_{u_n})]_{n_n} (\mu_1)^{n_1} (-1)^{e_1(r+s+p+q+f'+h'+1)n_1}}{[(b_k)]_{n_1} [(\alpha_{v_1})]_{n_1} [(\alpha_{v_2})]_{n_2} \dots [(\alpha_{v_n})]_{n_n} n_1! y^{e_1 n_1}} \\ & \times \frac{(\mu_2 x_2^{e_2})^{n_2} (-1)^{e_2(r+s+p+q+f'+h'+1)n_2}}{n_2!} \dots \frac{(\mu_n x_n^{e_n})^{n_n} (-1)^{e_n(r+s+p+q+f'+h'+1)n_n}}{n_n!} \end{aligned}$$

where j is non negative integer(3.3)

The single terminating factor $(-m+j) e_1 n_1 + e_2 n_2 + \dots + e_n n_n$

make all summation in (3.3) runs upto ∞

particular cases of (3.1)

1. Sylvester Polynomials

On setting $r = 0$ $s = p = q = \mu_1 = v_1 = k$

$h_1 = 1 = y = e = e_1 = \mu_1, a_1 = x$, we optain

$$\begin{aligned} & \phi_m^{(x)} \sum_{j=0}^m \binom{\frac{1}{2}}{j}_n \frac{[(f_h')]_m (-1)^j (2j+1)c_j(x^e)}{[(d_g')]_m (m-j)!(m+j+1)!} \\ & \times F \begin{bmatrix} -m+j-m-j-1, 1-\binom{d_g'}{j}-m, x; \\ -m, \frac{1}{2}-m, 1-\binom{f_h'}{j}-m \end{bmatrix} \end{aligned}$$

2. Bedient Polynomials :

On taking $s = 0 = u_n = v_n ; r = 1 = p = q = e = \mu_n , \mu_n = 4$

$e_n = 2 ; \mu = 2, D_1 = \infty + \alpha_1 A_1 = \alpha_1 ; c_1 = \alpha ,$ we get

$$G_m(\alpha, \beta, x) = \sum_{j=0}^m \frac{\left(\frac{1}{2}\right)_n [(f_h)]_m (-1)^j (2j+1)(\alpha)_m (\beta)_m 2^m c_j(x^e)}{[d_g]_m [(m-j)!(m+j+1)!(\alpha+\beta)_m]} \\ \times F\left[\begin{matrix} \Delta(2;-m+j), \Delta(2;-m-j-1), D(2;1-(d_2)-m), \Delta(2;1-\alpha+-\beta-m); \\ \Delta(2-m), \Delta(2;1/2-m), \Delta(2;1-(f_h)-m), \Delta(2;1+\infty-m), \Delta(2;1-\beta-m); \end{matrix} 1\right]$$

Similaly, specializing the parameters of (3.1) the polynomials of Sah, Khanna, Krawtchonk gonld, Humbrt, Lomenel pradhan, etc can also be deduced in terms of sister celin's polynomials.

References :

1. Askey. R.A. (1975); Theory and application of special function, Academic Press.
2. Burchnall, J.L. and Chaundy. T.W. (1941); Expansion of Appell's double hyper geomaric function (ii) 1941 Quart. J math, oxford, sec 12 p. 112-128.
3. Rail Ville, E.D. (1960) special functions, Macmillan, Co. New Yoak.
4. Shah, Manilal; Expansion formule for generalised hypergeometric polynomial in series of Jocobi Polynomials.
5. Rainville E.P. Special functics the in Macmillan Co., New York 1967.
6. Srivastava, B.M. and F.Singh; On some new generating relations, Ranchi, Univ. Math J.Vol. 5, 1974
7. Srivastava H.M. and Manocha, HL (1984); A tneatise on generating functions, Halsted press, John willy and Sons, New Yoark.
